1.2 Functions and Graphs

What you will learn about . . .

- Functions
- Domains and Ranges
- Viewing and Interpreting Graphs
- Even Functions and Odd Functions—Symmetry
- Functions Defined in Pieces
- Absolute Value Function
- Composite Functions

and why . . .

Functions and graphs form the basis for understanding mathematics and applications.

Figure 1.8 A "machine" diagram for a function.

Leonhard Euler (1707–1783)

Leonhard Euler, the dominant mathematical figure of his century and the most prolific mathematician ever, was also an astronomer, physicist, botanist, and chemist, and an expert in oriental languages. His work was the first to give the function concept the prominence that it has in mathematics today. Euler's collected books and papers fill 72 volumes. This does not count his enormous correspondence to approximately 300 addresses. His introductory algebra text, written originally in German (Euler was Swiss), is still available in English translation.

Functions

The values of one variable often depend on the values for another:

- The temperature at which water boils depends on elevation (the boiling point drops as you go up).
- The amount by which your savings will grow in a year depends on the interest rate offered by the bank.
- The area of a circle depends on the circle's radius.

In each of these examples, the value of one variable quantity depends on the value of another. For example, the boiling temperature of water, b, depends on the elevation, e; the amount of interest, I, depends on the interest rate, r. We call b and I **dependent variables** because they are determined by the values of the variables e and r on which they depend. The variables e and r are **independent variables.**

A rule that assigns to each element in one set a unique element in another set is called a *function.* The sets may be sets of any kind and do not have to be the same. A function is like a machine that assigns a unique output to every allowable input. The inputs make up the **domain** of the function; the outputs make up the **range** (Figure 1.8).

DEFINITION Function

A **function** from a set D to a set R is a rule that assigns a unique element in R to each element in D.

In this definition, D is the domain of the function and R is a set *containing* the range (Figure 1.9).

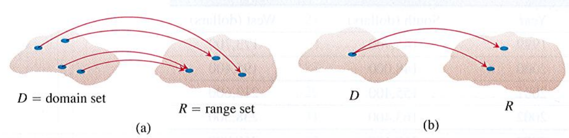

Figure 1.9 (a) A function from a set D to a set R. (b) *Not* a function. The assignment is not unique.

Euler invented a symbolic way to say "y is a function of x":

$$y = f(x),$$

which we read as "y equals f of x." This notation enables us to give different functions different names by changing the letters we use. To say that the boiling point of water is a function of elevation, we can write $b = f(e)$. To say that the area of a circle is a function of the circle's radius, we can write $A = A(r)$, giving the function the same name as the dependent variable.

The notation $y = f(x)$ gives a way to denote specific values of a function. The value of f at a can be written as $f(a)$, read "f of a."

EXAMPLE 1 The Circle-Area Function

Write a formula that expresses the area of a circle as a function of its radius. Use the formula to find the area of a circle of radius 2 in.

SOLUTION

If the radius of the circle is r, then the area $A(r)$ of the circle can be expressed as $A(r) = \pi r^2$. The area of a circle of radius 2 can be found by evaluating the function $A(r)$ at $r = 2$.

$$A(2) = \pi(2)^2 = 4\pi$$

The area of a circle of radius 2 is 4π in^2. ***Now Try Exercise 3.***

Domains and Ranges

In Example 1, the domain of the function is restricted by context: The independent variable is a radius and must be positive. When we define a function $y = f(x)$ with a formula and the domain is not stated explicitly or restricted by context, the domain is assumed to be the largest set of x-values for which the formula gives real y-values—the so-called **natural domain.** If we want to restrict the domain, we must say so. The domain of $y = x^2$ is understood to be the entire set of real numbers. We must write "$y = x^2, x > 0$" if we want to restrict the function to positive values of x.

The domains and ranges of many real-valued functions of a real variable are intervals or combinations of intervals. The intervals may be open, closed, or half-open (Figures 1.10 and 1.11) and finite or infinite (Figure 1.12).

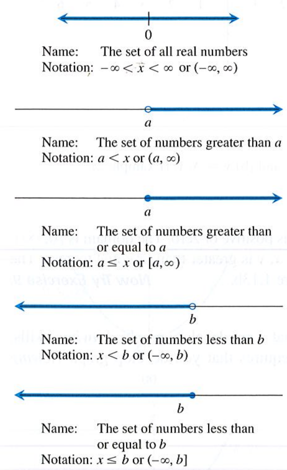

Figure 1.12 Infinite intervals—rays on the number line and the number line itself. The symbol ∞ (infinity) is used merely for convenience; it does not mean there is a number ∞.

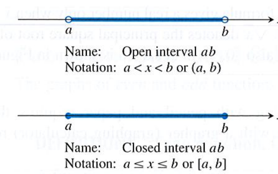

Figure 1.10 Open and closed finite intervals.

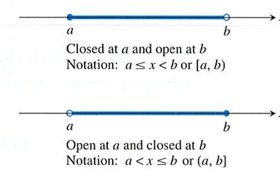

Figure 1.11 Half-open finite intervals.

The endpoints of an interval make up the interval's **boundary** and are called **boundary points.** The remaining points make up the interval's **interior** and are called **interior points. Closed intervals** contain their boundary points. **Open intervals** contain no boundary points. Every point of an open interval is an interior point of the interval.

Viewing and Interpreting Graphs

The points (x, y) in the plane whose coordinates are the input-output pairs of a function $y = f(x)$ make up the function's **graph.** The graph of the function $y = x + 2$, for example, is the set of points with coordinates (x, y) for which y equals $x + 2$.

EXAMPLE 2 Identifying Domain and Range of a Function

Identify the domain and range, and then sketch a graph of the function.

(a) $y = \dfrac{1}{x}$ **(b)** $y = \sqrt{x}$

SOLUTION

(a) The formula gives a real y-value for every real x-value except $x = 0$. (*We cannot divide any number by* 0.) The domain is $(-\infty, 0) \cup (0, \infty)$. The value y takes on every real number except $y = 0$. ($y = c \neq 0$ if $x = 1/c$). The range is also $(-\infty, 0) \cup (0, \infty)$. A sketch is shown in Figure 1.13a.

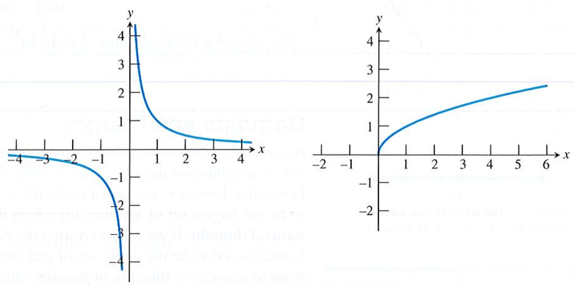

Figure 1.13 A sketch of the graph of (a) $y = 1/x$ and (b) $y = \sqrt{x}$. (Example 2)

(b) The formula gives a real number only when x is positive or zero. The domain is $[0, \infty)$. Because $\sqrt{x}$ denotes the principal square root of x, y is greater than or equal to zero. The range is also $[0, \infty)$. A sketch is shown in Figure 1.13b. ***Now Try Exercise 9.***

Graphing with pencil and paper requires that you develop graph *drawing* skills. Graphing with a grapher (graphing calculator) requires that you develop graph *viewing* skills.

Power Function

Any function that can be written in the form $f(x) = kx^a$, where k and a are nonzero constants, is a **power function.**

Graph Viewing Skills

1. Recognize that the graph is reasonable.
2. See all the important characteristics of the graph.
3. Interpret those characteristics.
4. Recognize grapher failure.

Being able to recognize that a graph is reasonable comes with experience. You need to know the basic functions, their graphs, and how changes in their equations affect the graphs.

Grapher failure occurs when the graph produced by a grapher is less than precise—or even incorrect—usually due to the limitations of the screen resolution of the grapher.

EXAMPLE 3 Identifying Domain and Range of a Function

Use a grapher to identify the domain and range, and then draw a graph of the function.

(a) $y = \sqrt{4 - x^2}$ **(b)** $y = x^{2/3}$

SOLUTION

(a) Figure 1.14a shows a graph of the function for $-4.7 \le x \le 4.7$ and $-3.1 \le y \le 3.1$, that is, the viewing window $[-4.7, 4.7]$ by $[-3.1, 3.1]$, with x-scale $= y$-scale $= 1$. The graph appears to be the upper half of a circle. The domain appears to be $[-2, 2]$. This observation is correct because we must have $4 - x^2 \ge 0$, or equivalently, $-2 \le x \le 2$. The range appears to be $[0, 2]$, which can also be verified algebraically.

Graphing $y = x^{2/3}$—Possible Grapher Failure

On some graphing calculators you need to enter this function as $y = (x^2)^{1/3}$ or $y = (x^{1/3})^2$ to obtain a correct graph. Try graphing this function on your grapher.

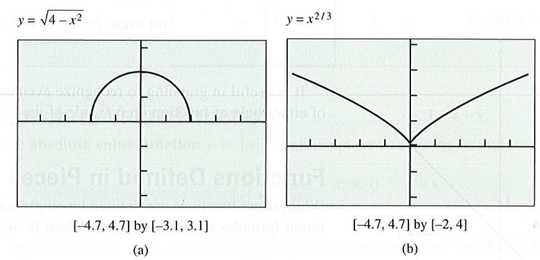

Figure 1.14 The graph of (a) $y = \sqrt{4 - x^2}$ and (b) $y = x^{2/3}$. (Example 3)

(b) Figure 1.14b shows a graph of the function in the viewing window $[-4.7, 4.7]$ by $[-2, 4]$, with x-scale $= y$-scale $= 1$. The domain appears to be $(-\infty, \infty)$, which we can verify by observing that $x^{2/3} = (\sqrt[3]{x})^2$. Also the range is $[0, \infty)$ by the same observation. ***Now Try Exercise 15.***

Even Functions and Odd Functions—Symmetry

The graphs of *even* and *odd* functions have important symmetry properties.

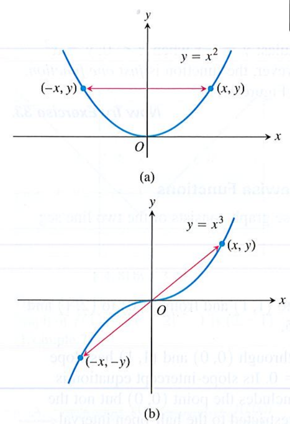

Figure 1.15 (a) The graph of $y = x^2$ (an even function) is symmetric about the y-axis. (b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

DEFINITIONS Even Function, Odd Function

A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$,

odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.

The names even and odd come from powers of x. If y is an even power of x, as in $y = x^2$ or $y = x^4$, it is an even function of x (because $(-x)^2 = x^2$ and $(-x)^4 = x^4$). If y is an odd power of x, as in $y = x$ or $y = x^3$, it is an odd function of x (because $(-x)^1 = -x$ and $(-x)^3 = -x^3$).

The graph of an even function is **symmetric about the y-axis.** Since $f(-x) = f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, y)$ lies on the graph (Figure 1.15a).

The graph of an odd function is **symmetric about the origin.** Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, -y)$ lies on the graph (Figure 1.15b).

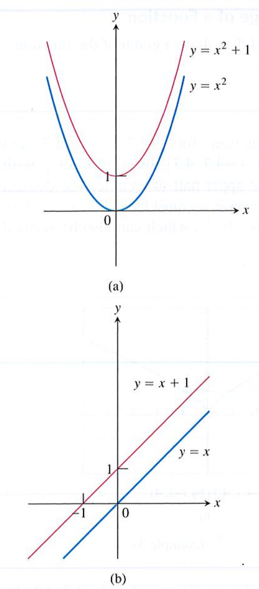

Figure 1.16 (a) When we add the constant term 1 to the function $y = x^2$, the resulting function $y = x^2 + 1$ is still even and its graph is still symmetric about the y-axis. (b) When we add the constant term 1 to the function $y = x$, the resulting function $y = x + 1$ is no longer odd. The symmetry about the origin is lost. (Example 4)

Equivalently, a graph is symmetric about the origin if a rotation of 180° about the origin leaves the graph unchanged.

EXAMPLE 4 Recognizing Even and Odd Functions

$f(x) = x^2$ Even function: $(-x)^2 = x^2$ for all x; symmetry about y-axis.

$f(x) = x^2 + 1$ Even function: $(-x)^2 + 1 = x^2 + 1$ for all x; symmetry about y-axis (Figure 1.16a).

$f(x) = x$ Odd function: $(-x) = -x$ for all x; symmetry about the origin.

$f(x) = x + 1$ Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal.

Not even: $(-x) + 1 \neq x + 1$ for all $x \neq 0$ (Figure 1.16b).

Now Try Exercises 21 and 23.

It is useful in graphing to recognize even and odd functions. Once we know the graph of either type of function on one side of the y-axis, we know its graph on both sides.

Functions Defined in Pieces

While some functions are defined by single formulas, others are defined by applying different formulas to different parts of their domains.

EXAMPLE 5 Graphing Piecewise-Defined Functions

Graph $y = f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1. \end{cases}$

SOLUTION

The values of f are given by three separate formulas: $y = -x$ when $x < 0$, $y = x^2$ when $0 \le x \le 1$, and $y = 1$ when $x > 1$. However, the function is *just one function*, whose domain is the entire set of real numbers (Figure 1.17).

Now Try Exercise 33.

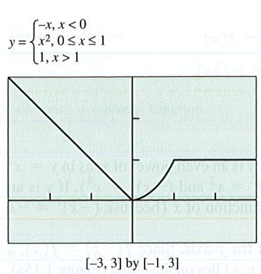

Figure 1.17 The graph of a piecewise-defined function. (Example 5)

EXAMPLE 6 Writing Formulas for Piecewise Functions

Write a formula for the function $y = f(x)$ whose graph consists of the two line segments in Figure 1.18.

SOLUTION

We find formulas for the segments from $(0, 0)$ to $(1, 1)$ and from $(1, 0)$ to $(2, 1)$ and piece them together in the manner of Example 5.

Segment from (0, 0) to (1, 1) The line through $(0, 0)$ and $(1, 1)$ has slope $m = (1 - 0)/(1 - 0) = 1$ and y-intercept $b = 0$. Its slope-intercept equation is $y = x$. The segment from $(0, 0)$ to $(1, 1)$ that includes the point $(0, 0)$ but not the point $(1, 1)$ is the graph of the function $y = x$ restricted to the half-open interval $0 \le x < 1$, namely,

$$y = x, \quad 0 \le x < 1.$$

continued

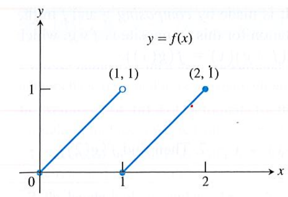

Figure 1.18 The segment on the left contains $(0, 0)$ but not $(1, 1)$. The segment on the right contains both of its endpoints. (Example 6)

Segment from (1, 0) to (2, 1) The line through $(1, 0)$ and $(2, 1)$ has slope $m = (1 - 0)/(2 - 1) = 1$ and passes through the point $(1, 0)$. The corresponding point-slope equation for the line is

$$y = 1(x - 1) + 0, \quad \text{or} \quad y = x - 1.$$

The segment from $(1, 0)$ to $(2, 1)$ that includes both endpoints is the graph of $y = x - 1$ restricted to the closed interval $1 \le x \le 2$, namely,

$$y = x - 1, \quad 1 \le x \le 2.$$

Piecewise Formula Combining the formulas for the two pieces of the graph, we obtain

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ x - 1, & 1 \le x \le 2. \end{cases}$$

Now Try Exercise 43.

Absolute Value Function

The **absolute value function** $y = |x|$ is defined piecewise by the formula

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \ge 0. \end{cases}$$

The function is even, and its graph (Figure 1.19) is symmetric about the y-axis.

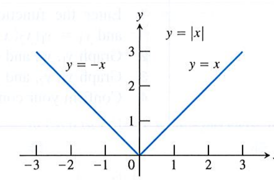

Figure 1.19 The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

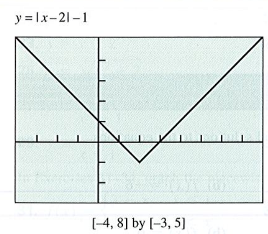

Figure 1.20 The lowest point of the graph of $f(x) = |x - 2| - 1$ is $(2, -1)$. (Example 7)

EXAMPLE 7 Using Transformations

Draw the graph of $f(x) = |x - 2| - 1$. Then find the domain and range.

SOLUTION

The graph of f is the graph of the absolute value function shifted 2 units horizontally to the right and 1 unit vertically downward (Figure 1.20). The domain of f is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

Now Try Exercise 49.

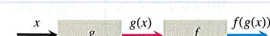

Figure 1.21 Two functions can be composed when a portion of the range of the first lies in the domain of the second.

Composite Functions

Suppose that some of the outputs of a function g can be used as inputs of a function f. We can then link g and f to form a new function whose inputs x are inputs of g and whose outputs are the numbers $f(g(x))$, as in Figure 1.21. We say that the function $f(g(x))$

(read "f of g of x") is the **composite of g and f**. It is made by *composing* g and f in the order of first g, then f. The usual "stand-alone" notation for this composite is $f \circ g$, which is read as "f of g." Thus, the value of $f \circ g$ at x is $(f \circ g)(x) = f(g(x))$.

EXAMPLE 8 Composing Functions

Find a formula for $f(g(x))$ if $g(x) = x^2$ and $f(x) = x - 7$. Then find $f(g(2))$.

SOLUTION

To find $f(g(x))$, we replace x in the formula $f(x) = x - 7$ by the expression given for $g(x)$.

$$f(x) = x - 7$$

$$f(g(x)) = g(x) - 7 = x^2 - 7$$

We then find the value of $f(g(2))$ by substituting 2 for x.

$$f(g(2)) = (2)^2 - 7 = -3$$

Now Try Exercise 51.

EXPLORATION 1 Composing Functions

Some graphers allow a function such as y_1 to be used as the independent variable of another function. With such a grapher, we can compose functions.

1. Enter the functions $y_1 = f(x) = 4 - x^2$, $y_2 = g(x) = \sqrt{x}$, $y_3 = y_2(y_1(x))$, and $y_4 = y_1(y_2(x))$. Which of y_3 and y_4 corresponds to $f \circ g$? to $g \circ f$?
2. Graph y_1, y_2, and y_3 and make conjectures about the domain and range of y_3.
3. Graph y_1, y_2, and y_4 and make conjectures about the domain and range of y_4.
4. Confirm your conjectures algebraically by finding formulas for y_3 and y_4.

Quick Review 1.2 *(For help, go to Appendix A1 and Section 1.2.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

In Exercises 1–6, solve for x.

1. $3x - 1 \le 5x + 3$
2. $x(x - 2) > 0$
3. $|x - 3| \le 4$
4. $|x - 2| \ge 5$
5. $x^2 < 16$
6. $9 - x^2 \ge 0$

In Exercises 7 and 8, describe how the graph of f can be transformed to the graph of g.

7. $f(x) = x^2$, $g(x) = (x + 2)^2 - 3$
8. $f(x) = |x|$, $g(x) = |x - 5| + 2$

In Exercises 9–12, find all real solutions to the equations.

9. $f(x) = x^2 - 5$
   (a) $f(x) = 4$ (b) $f(x) = -6$
10. $f(x) = 1/x$
   (a) $f(x) = -5$ (b) $f(x) = 0$
11. $f(x) = \sqrt{x + 7}$
   (a) $f(x) = 4$ (b) $f(x) = 1$
12. $f(x) = \sqrt[3]{x - 1}$
   (a) $f(x) = -2$ (b) $f(x) = 3$

Section 1.2 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

In Exercises 1–4, **(a)** write a formula for the function and **(b)** use the formula to find the indicated value of the function.

1. the area A of a circle as a function of its diameter d; the area of a circle of diameter 4 in.
2. the height h of an equilateral triangle as a function of its side length s; the height of an equilateral triangle of side length 3 m
3. the surface area S of a cube as a function of the length of the cube's edge e; the surface area of a cube of edge length 5 ft
4. the volume V of a sphere as a function of the sphere's radius r; the volume of a sphere of radius 3 cm

In Exercises 5–12, **(a)** identify the domain and range and **(b)** sketch the graph of the function.

5. $y = 4 - x^2$
6. $y = x^2 - 9$
7. $y = 2 + \sqrt{x - 1}$
8. $y = -\sqrt{-x}$
9. $y = \dfrac{1}{x - 2}$
10. $y = \sqrt[4]{-x}$
11. $y = 1 + \dfrac{1}{x}$
12. $y = 1 + \dfrac{1}{x^2}$

In Exercises 13–20, use a grapher to **(a)** identify the domain and range and **(b)** draw the graph of the function.

13. $y = \sqrt[3]{x}$
14. $y = 2\sqrt{3 - x}$
15. $y = \sqrt[3]{1 - x^2}$
16. $y = \sqrt{9 - x^2}$
17. $y = x^{2/5}$
18. $y = x^{3/2}$
19. $y = \sqrt[3]{x - 3}$
20. $y = \dfrac{1}{\sqrt{4 - x^2}}$

In Exercises 21–30, determine whether the function is even, odd, or neither. Try to answer without writing anything (except the answer).

21. $y = x^4$
22. $y = x + x^2$
23. $y = x + 2$
24. $y = x^2 - 3$
25. $y = \sqrt{x^2 + 2}$
26. $y = x + x^3$
27. $y = \dfrac{x^3}{x^2 - 1}$
28. $y = \sqrt[3]{2 - x}$
29. $y = \dfrac{1}{x - 1}$
30. $y = \dfrac{1}{x^2 - 1}$

In Exercises 31–34, graph the piecewise-defined functions.

31. $f(x) = \begin{cases} 3 - x, & x \le 1 \\ 2x, & 1 < x \end{cases}$
32. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$
33. $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ (3/2)x + 3/2, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$
34. $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$

35. **Writing to Learn** The *vertical line test* to determine whether a curve is the graph of a function states: If every vertical line in the xy-plane intersects a given curve in at most one point, then the curve is the graph of a function. Explain why this is true.
36. **Writing to Learn** For a curve to be *symmetric about the x-axis,* the point (x, y) must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the x-axis is not the graph of a function, unless the function is $y = 0$.

In Exercises 37–40, use the vertical line test (see Exercise 35) to determine whether the curve is the graph of a function.

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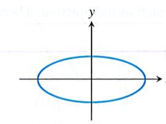

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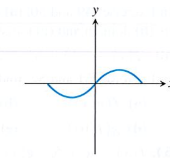

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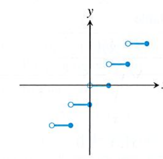

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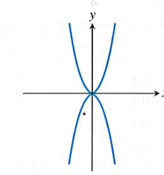

In Exercises 41–48, write a piecewise formula for the function.

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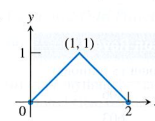

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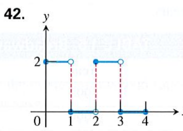
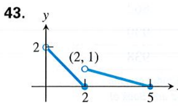


43.


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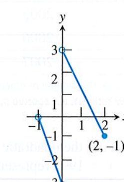

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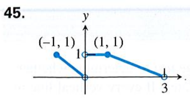

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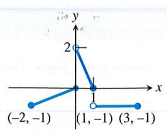

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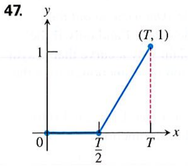

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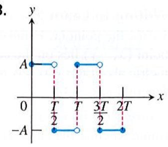


In Exercises 49 and 50, **(a)** draw the graph of the function. Then find its **(b)** domain and **(c)** range.

49. $f(x) = -|3 - x| + 2$ **50.** $f(x) = 2|x + 4| - 3$

In Exercises 51 and 52, find

(a) $f(g(x))$ **(b)** $g(f(x))$ **(c)** $f(g(0))$

(d) $g(f(0))$ **(e)** $g(g(-2))$ **(f)** $f(f(x))$

51. $f(x) = x + 5, \quad g(x) = x^2 - 3$

52. $f(x) = x + 1, \quad g(x) = x - 1$

53. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	?	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
(b)	?	$1 + 1/x$	x
(c)	$1/x$	?	x
(d)	$\sqrt{x}$	?	$\|x\|, x \geq 0$

54. *Broadway Season Statistics* Table 1.5 shows the gross revenue for the Broadway season in millions of dollars for several years.

TABLE 1.5 Broadway Season Revenue

Year	Amount ($ millions)
1994	406
1999	603
2004	769
2005	862
2006	939
2007	938

Source: The League of American Theatres and Producers, Inc., New York, NY, as reported in *The World Almanac and Book of Facts, 2009.*

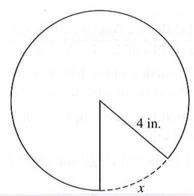
(a) Find the quadratic regression for the data in Table 1.5. Let $x = 1990$ represent 1990, $x = 1991$ represent 1991, and so forth.

(b) Superimpose the graph of the quadratic regression equation on a scatter plot of the data.

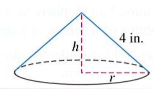
(c) Use the quadratic regression to predict the amount of revenue in 2012.

(d) Now find the linear regression for the data and use it to predict the amount of revenue in 2012.

55. *The Cone Problem* Begin with a circular piece of paper with a 4-in. radius as shown in (a). Cut out a sector with an arc length of x. Join the two edges of the remaining portion to form a cone with radius r and height h, as shown in (b).



(a)



(b)

(a) Explain why the circumference of the base of the cone is $8\pi - x$.

(b) Express the radius r as a function of x.

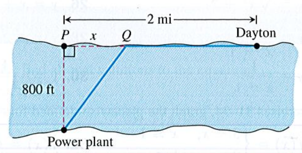
(c) Express the height h as a function of x.

(d) Express the volume V of the cone as a function of x.

56. *Industrial Costs* Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.

(a) Suppose that the cable goes from the plant to a point Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance x.

(b) Generate a table of values to determine if the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from point P.



(Not to scale)

Standardized Test Questions

57. True or False The function $f(x) = x^4 + x^2 + x$ is an even function. Justify your answer.

58. True or False The function $f(x) = x^{-3}$ is an odd function. Justify your answer.

59. Multiple Choice Which of the following gives the domain of $f(x) = \dfrac{x}{\sqrt{9 - x^2}}$?

(A) $x \neq \pm 3$ **(B)** $(-3, 3)$ **(C)** $[-3, 3]$

(D) $(-\infty, -3) \cup (3, \infty)$ **(E)** $(3, \infty)$

60. Multiple Choice Which of the following gives the range of $f(x) = 1 + \dfrac{1}{x - 1}$?

(A) $(-\infty, 1) \cup (1, \infty)$ **(B)** $x \neq 1$ **(C)** all real numbers

(D) $(-\infty, 0) \cup (0, \infty)$ **(E)** $x \neq 0$

61. Multiple Choice If $f(x) = 2x - 1$ and $g(x) = x + 3$, which of the following gives $(f \circ g)(2)$?

(A) 2 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 10

62. Multiple Choice The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width?

(A) $A(W) = 3W$ **(B)** $A(W) = \frac{1}{2}W^2$ **(C)** $A(W) = 2W^2$

(D) $A(W) = W^2 + 2W$ **(E)** $A(W) = W^2 - 2W$

Explorations

In Exercises 63–66, **(a)** graph $f \circ g$ and $g \circ f$ and make a conjecture about the domain and range of each function. **(b)** Then confirm your conjectures by finding formulas for $f \circ g$ and $g \circ f$.

63. $f(x) = x - 7, \quad g(x) = \sqrt{x}$

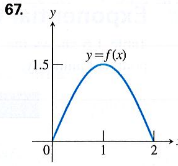
64. $f(x) = 1 - x^2, \quad g(x) = \sqrt{x}$

65. $f(x) = x^2 - 3, \quad g(x) = \sqrt{x + 2}$

66. $f(x) = \dfrac{2x - 1}{x + 3}, \quad g(x) = \dfrac{3x + 1}{2 - x}$

Group Activity In Exercises 67–70, a portion of the graph of a function defined on $[-2, 2]$ is shown. Complete each graph assuming that the graph is **(a)** even, **(b)** odd.

67.



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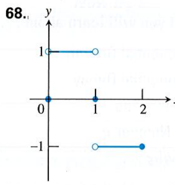

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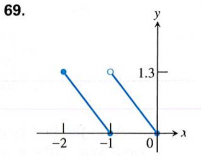
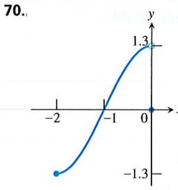


70.



Extending the Ideas

71. Enter $y_1 = \sqrt{x}$, $y_2 = \sqrt{1 - x}$ and $y_3 = y_1 + y_2$ on your grapher.

(a) Graph y_3 in $[-3, 3]$ by $[-1, 3]$.

(b) Compare the domain of the graph of y_3 with the domains of the graphs of y_1 and y_2.

(c) Replace y_3 by

$y_1 - y_2, \quad y_2 - y_1, \quad y_1 \cdot y_2, \quad y_1/y_2, \quad \text{and} \quad y_2/y_1,$

in turn, and repeat the comparison of part (b).

(d) Based on your observations in (b) and (c), what would you conjecture about the domains of sums, differences, products, and quotients of functions?

72. *Even and Odd Functions*

(a) Must the product of two even functions always be even? Give reasons for your answer.

(b) Can anything be said about the product of two odd functions? Give reasons for your answer.